Problem Solving Session

Be prepared to discuss the following problems in class on Thursday, **October 24**.

1. Given six people, show that either three are mutual friends, or three are complete strangers to one another. (Assume that friendship is mutual; i.e., if you are my friend, then I must be your friend.)

2. A group of people play a round-robin chess tournament, which means that everyone plays a game with everyone else exactly once (chess is a one-on-one game, not a team sport). There are no draws. Prove that it is always possible to line up the players in such a way that the first player beat the second, who beat the third, etc. down to the last player. Hence it is always possible to declare not only a winner, but a meaningful ranking of all the players. Give a graph theoretic statement of the above. Must this ranking be unique?

3. Let $a_0 = 1$, $a_1 = 1$, $a_2 = 2$, and for $n \geq 3$ define a_n to be the last digit of the sum of the preceding three terms in the sequence. Thus the first few terms of this sequence of digits are (in concatenated form) 1124734419447... Determine whether or not the string 1001 occurs somewhere in this sequence.