

Problem Solving Session

*Be prepared to discuss the following problems in class on Thursday, **October 31**.*

1. A bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint sets U and V such that every edge in the graph connects a vertex in the set U to a vertex in the set V . Prove that a finite graph is bipartite if and only if it contains no cycles of odd length.
2. A complete graph is a graph in which each pair of graph vertices is connected by an edge. Prove that a complete graph with n vertices contains $n(n - 1)/2$ edges.
3. Suppose $P(x)$ is a polynomial with integer coefficients such that none of the values $P(1), \dots, P(2009)$ is divisible by 2009. Prove that $P(n) \neq 0$ for all integers n .