

Problem Solving Session

*Be prepared to discuss the following problems in class on Thursday, **September 19**.*

1. For each positive integer n , find positive integer solutions x_1, x_2, \dots, x_n to the equation

$$\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n} + \frac{1}{x_1 x_2 \dots x_n} = 1.$$

2. 1. Consider the following two operations on two piles of coins.

(A) Add one coin to each pile.

(B) Double the number of coins in either pile and reduce the other by half.

Operation (B) is permitted only when it will result in whole numbers of coins, for example, $(3, 8) \rightarrow (6, 4)$ or $(100, 6) \rightarrow (50, 12)$. Beginning with two coins in one pile and three in the other show that no sequence of the operations (A) and (B) will result in the piles being of equal size.

3. Show that any integer amount $n \geq 60$ cents can be changed using 6-cent and 11-cent coins. Equivalently, any integer $n \geq 60$ can be expressed as $n = 6a + 11b$, where a and b are nonnegative integers.