Homework Assignment

Due Nov 11.

1. Give an example of a decomposition of $\mathbb{R}^2$ into a disjoint union of immersed, closed in $\mathbb{R}^2$, smooth submanifolds of dimension 1 which is not a foliation.

2. Prove that the Reeb foliation on S^3 cannot be given by a submersion. *(Hint: show that if a level set $f^{-1}(a)$ of a submersion f is compact, then for a' sufficiently close to a, the set $f^{-1}(a')$ is also compact.)*

3. Prove the following: a foliation $\mathcal{F}$ of dimension 1 on a surface S is orientable if and only if it is generated by integral curves of a nonsingular vector field on S.

4. On $\mathbb{R}^2$ consider the foliation given by level sets of the function $f(x,y) = y - \alpha x$, where $\alpha \in \mathbb{R}$ is fixed. Prove that this foliation is invariant under the action of $(\mathbb{Z} \times \mathbb{Z}, +)$. Conclude that this gives rize to a foliation $\mathcal{F}_\alpha$ on the torus $T^2 = \mathbb{R}^2/(\mathbb{Z} \times \mathbb{Z})$. Show that if $\alpha \in \mathbb{Q}$, then all leaves of $\mathcal{F}_\alpha$ are circles, and if $\alpha \in \mathbb{R} \setminus \mathbb{Q}$, then all leaves are lines that are dense in T^2.