MATH 9312 SUMMER 2013

HOMEWORK ASSIGNMENT 1. DUE MAY 21.

1.1. Suppose that a function ϕ is real analytic on $\mathbb{R}^n$ and is a test function. Prove that $\phi \equiv 0$.

1.2. Suppose that $\phi \in \mathcal{D}(\Omega)$ for a domain $\Omega \subset \mathbb{R}^n$. Let $K = \operatorname{supp} \phi$. Determine $\operatorname{supp} \phi_\epsilon$, where ϕ_ϵ is the regularization of ϕ.

1.3. Suppose that f and g are test functions on $\mathbb{R}^n$. Prove that $f * g = g * f$.

1.4. Suppose (f_k) is a sequence of continuous functions converging to a function f in $\mathcal{C}(\mathbb{R})$. Prove that (f_k) converges to f in $\mathcal{D}'(\mathbb{R})$.

1.5. Prove that
$$\lim_{n \to \infty} \sin(nx) = 0$$
in $\mathcal{D}'(\mathbb{R})$, but
$$\lim_{n \to \infty} \sin^2(nx) \neq 0.$$
Conclude that multiplication of distributions is not a continuous operation even where it is defined.