MATH 9312 SUMMER 2013

HOMEWORK ASSIGNMENT 3. DUE JUNE 4.

3.1. Let $f \in \mathcal{D}'(\mathbb{R})$ be such that $f(x+h) = f(x)$ for every positive h. Show that f is constant.

3.2. Prove that f given by

$$\langle f, \phi \rangle = \sum_{n=0}^{\infty} \phi^{(n)}(n), \quad \phi \in \mathcal{D}(\mathbb{R})$$

is a distribution of infinite order of singularity.

3.3. For $f, g \in \mathcal{D}'(\mathbb{R})$ prove that $f * g$ is a well-defined distribution if $\operatorname{supp} f \subset (-\infty, a)$ and $\operatorname{supp} g \subset (b, \infty)$ for some $a, b \in \mathbb{R}$.

3.4. Let f be a distribution with compact support and let P be a polynomial. Show that $P * f$ is a polynomial.