MATH 9312 SUMMER 2013

HOMEWORK ASSIGNMENT 4. DUE JUNE 25.

4.1. Find a generalized solution to the differential equation on $\mathbb{R}$ of the form

$$\frac{d^2}{dx^2}u = \chi_{[0,1]},$$

where $\chi_{[0,1]}$ is the characteristic function of the interval $[0,1] \subset \mathbb{R}$. Is the solution unique?

4.2. Let a, b be real numbers. Find the fundamental solutions for the differential operator

$$D = a\frac{\partial^2}{\partial x_1^2} + b\frac{\partial^2}{\partial x_2^2}$$

on $\mathbb{R}^2$ with the standard coordinates (x_1, x_2).

4.3. Prove that if a C^2-smooth function $u(x)$ on $\mathbb{R}^n$ satisfies the inequality

$$u(a) = \frac{1}{\sigma_n R^{n-1}} \int_{B(a,R)} u(x) dS$$

for all balls $B(a,R) \subset \mathbb{R}^n$, then $u(x)$ is a harmonic function, i.e., $\Delta u = 0$. Generalize this result to subharmonic functions.

4.4. Let $H^1(\Omega)$ be the space of $L^2(\Omega)$ functions whose first order partial derivatives in the distributional sense also belong to $L^2(\Omega)$. Prove that $H^1(\Omega)$ is a Hilbert space with respect to the scalar product

$$[u,v]_{H^1(\Omega)} = \int_\Omega u\, v\, dx + \int_\Omega (\nabla u, \nabla v) dx.$$

4.5. Find the general form of a distribution $f \in \mathcal{D}'(\mathbb{R}^n)$ whose support is a finite set of points in $\mathbb{R}^n$.