MATHEMATICS 9302: RIEMANN SURFACES. SUMMER 2014

HOMEWORK ASSIGNMENT 1.

Due May 27.

1.1. What is the smallest number of coordinate charts necessary to make a torus into a Riemann surface?

1.2. Prove Euler's identity: If $P(z_0, z_1, z_2)$ is a homogeneous polynomial of degree $d > 0$, then

$$\sum_{j=0}^{2} z_j \frac{\partial P(z)}{\partial z_j} = d \cdot P(z).$$

1.3. Algebraic curves of the form $F_n = \{z_1^n + z_2^n = z_0^n\}$, $n \in \mathbb{N}$, are called the Fermat curves. Prove that F_n are smooth in $\mathbb{P}_2$. Find all complex lines $L \subset \mathbb{C}^2$, such that their closures in $\mathbb{P}_2$ are tangent to F_n at points in $\mathbb{P}_2 \setminus \mathbb{C}^2$.

1.4. Discuss all singularities of the function

$$f(z) = \frac{\sin\sqrt{z}}{\sqrt{z}},$$

for $z \in \mathbb{P}_1 = \mathbb{C} \cup \{\infty\}$.

1.5. Let X be a Riemann surface, and let f be a meromorphic function on X. Suppose that $p \in X$ is a pole of f. Prove that the order of the pole at p is well-defined, i.e., is independent of the choice of a coordinate chart, but the residue of f at p is not.