MATHEMATICS 9302: RIEMANN SURFACES. SUMMER 2014

HOMEWORK ASSIGNMENT 2.

Due June 5.

2.1. Prove the change of coordinates formula for differential 1-forms: if in a coordinate system (U, x) a form is given by $\alpha = \sum_{j=1}^{n} c_j(x) dx_j$, then in the coordinate system $(\tilde{U}, \tilde{x})$ the same form is given by

$$\alpha = \sum_{\nu} \left(\sum_{j} c_j \frac{\partial x_j}{\partial \tilde{x}_\nu} \right) d\tilde{x}_\nu.$$

2.2. Compute the Euler characteristic of the Fermat curves

$$F_n = \{z_1^n + z_2^n = z_0^n\}$$

(see Problem 1.3)

2.3. Prove that for any smooth function f on a Riemann surface X we have

$$d(df) = \partial(\partial f) = \overline{\partial}(\overline{\partial} f) = 0,$$

where $d = \partial + \overline{\partial}$ is the operator of exterior differentiation.

2.4. Prove that 1-form α on a Riemann surface X is exact if and only if $\int_\gamma \alpha = 0$ for any 1-cycle $\gamma \subset X$.

2.5. Show that there exists a smooth vector field on $\mathbb{P}_1 = \mathbb{C} \cup \{\infty\}$ that vanishes at exactly one point. (Hint: use stereographic projection.)