MATHEMATICS 9302: RIEMANN SURFACES. SUMMER 2014

HOMEWORK ASSIGNMENT 3.

Due June 19.

3.1. Prove that $H^0(\mathbb{P}_1, \mathbb{C}) \cong H^2(\mathbb{P}_1, \mathbb{C}) \cong \mathbb{C}$, and that $H^1(\mathbb{P}_1, \mathbb{C}) \cong 0$.

3.2. Show that harmonic functions are correctly defined on any Riemann surface. Show that any harmonic function on a compact Riemann surface is constant.

3.3. Show that if $h(z)$ is a complex-valued harmonic function such that $zh(z)$ is also harmonic, then $h(z)$ is holomorphic.

3.4. Show that a complex-valued function $h(z)$ on a star-shaped domain $D \subset \mathbb{C}$ is harmonic if and only if $h(z) = f(z) + \overline{g(z)}$, where $f(z)$ and $g(z)$ are holomorphic functions on D.

3.5. Let $u(z)$ be a continuous function from a domain $D \subset \mathbb{C}$ to $[-\infty, \infty)$. Suppose $u_n(z)$ is a decreasing sequence of subharmonic functions on D such that $u_n(z) \to u(z)$ for all $z \in D$. Show that $u(z)$ is subharmonic.