# MATHEMATICS 9302: RIEMANN SURFACES. SUMMER 2014

HOMEWORK ASSIGNMENT 4.

Due July 3.

4.1. Suppose that $F : X \to Y$ is a nonconstant holomorphic map between Riemann surfaces.
(i) Show that $F(X)$ is an open subset of $Y$.
(ii) Show that if $X$ is compact, then $F(X) = Y$.
(iii) Use (ii) to prove the Fundamental Theorem of Algebra.

4.2. Let $\Omega$ be the annulus $\{a < |z| < b\}$ in $\mathbb{C}$. Let $\phi(z)$ be the function which is equal to a constant $\alpha$ on $\{|z| = a\}$, and to a constant $\beta$ on $\{|z| = b\}$. Find the Perron function for the family $E(\phi)$.

4.3. Find Green's function for the unit disc $\mathbb{D} = \{z : |z| < 1\}$ with the pole at a point $\zeta \in \mathbb{D}$.

4.4. Let $X$ be a Riemann surface, and let $u : X \to \mathbb{R}$ be a $C^2$-smooth function.
(i) Show that the property $\Delta u \geq 0$ is independent of the choice of a coordinate chart.
(ii) Prove that $u$ is subharmonic if and only if $\Delta u \geq 0$.

4.5. Show that if Green's function exists on a Riemann surface $X$, and $Y \subset X$ is a domain, then Green's function exists on $Y$, and $g_Y \leq g_X$.