CALCULUS 1501 WINTER 2010

HOMEWORK ASSIGNMENT 10.

Due April 5.

10.1. Find the Maclaurin series for $f(x)$. Find the radius of convergence of the series, and show, using Lagrange's remainder theorem that the series converges to $f(x)$.

(i) $f(x) = xe^{2x}$.

(ii) $f(x) = \sin x^2$.

10.2. Find the first 3 terms of the Taylor series of $f(x)$ centred at a:

(i) $f(x) = \sec x$ at $a = \frac{\pi}{4}$.

(ii) $f(x) = \tan^{-1} x$ at $a = 1$.

10.3. Find the Maclaurin series for $f(x) = \dfrac{x^2}{\sqrt{1+x}}$.

10.4. Approximate the function $f(x) = x^x$ at $a = 1$ by a Taylor polynomial of degree 2. How good is your approximation at $x = 1.1$?

10.5. Find the Cartesian equation of the curve which is given by a parametric equation

$$x = 2\cos\theta, \quad y = 3\sin\theta, \quad \theta \in (-\pi, \pi).$$

Sketch the curve indicating with an arrow the direction in which the curve is traced as the parameter increases.

10.6. Evaluate the integral

$$\int_0^1 \frac{\ln(1-x)}{x} dx.$$

Hint: Use Taylor series expansion and the identity $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.