

CALCULUS 1501 WINTER 2010

HOMEWORK ASSIGNMENT 4.

Due February 5.

4.1. Evaluate

$$\int_0^1 \frac{x \, dx}{\sqrt{2-x^4}}$$

4.2. Evaluate

$$\int \frac{y^3 + 1}{y^3 - y^2} \, dy$$

4.3. Determine whether the following improper integrals converge or diverge. Evaluate the integral if it converges.

- (i) $\int_2^\infty \frac{dx}{x^2 - 1}$
- (ii) $\int_0^\infty \cos 2x \, dx$
- (iii) $\int_1^\infty \frac{\arctan x}{x^2} \, dx$

4.4. (i) Investigate the convergence of the integral

$$\int_0^1 \frac{dx}{x^p}$$

for different values of $p > 0$.

(ii) Use (i) and properties of $\int_1^\infty \frac{dx}{x^p}$ to determine convergence of

$$\int_0^\infty \frac{dx}{x^p}$$

4.5. Determine whether the improper integral

$$\int_1^\infty \frac{\sin^2 3x}{\sqrt[3]{x^4 + 1}} \, dx$$

converges or diverges. Do not evaluate the integral if it converges.

4.6. Use the identity

$$\int_{-\infty}^\infty e^{-x^2} \, dx = \sqrt{\pi}$$

to evaluate $\Gamma(1/2)$ and $\Gamma(5/2)$. For relevant definitions see [Lecture 3](#) of the Course Notes.