CALCULUS 1501 WINTER 2010

HOMEWORK ASSIGNMENT 8.

Due March 12.

8.1. Determine whether the series converges absolutely, conditionally, or diverges.

(i) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+10}$

(ii) $\sum_{n=1}^{\infty} n^5\left(\frac{2-3n}{4n+3}\right)^n$

(iii) $\sum_{n=1}^{\infty}\frac{\sin 5n}{n^5}$.

8.2. Let $\{f_n\}$ be the Fibonacci sequence, given by $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$, for $n > 2$. Use Problem 5.3 (Homework 5) to assess the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{f_n}$$

8.3. Determine the convergence of the series

$$\frac{2}{5}+\frac{2\cdot 6}{5\cdot 8}+\frac{2\cdot 6\cdot 10}{5\cdot 8\cdot 11}+\frac{2\cdot 6\cdot 10\cdot 14}{5\cdot 8\cdot 11\cdot 14}+\cdots$$

8.4. (i) Show that the following series converges.

$$\sum_{n=1}^{\infty}\frac{a^n}{n!},\quad a > 0.$$

(ii) Explain how to use the result of part (i) to prove that $\lim_{n\to\infty}\frac{a^n}{n!} = 0$ for all $a > 0$.

8.5. Determine whether the series below converges absolutely, conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n\, n!}{n^n}$$

Hint: Recall that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

8.6. Assess the convergence of the following alternating series

$$\sum_{n=2}^{\infty}\left(\frac{1}{\sqrt{n}-1}-\frac{1}{\sqrt{n}+1}\right).$$