CALCULUS 1501 WINTER 2010

HOMEWORK ASSIGNMENT 9.

Due March 26.

9.1. Find the radius and the interval of convergence of the following power series

(i) $\sum_{n=0}^{\infty} 5^n x^{3n}$.

(ii) $\sum_{n=0}^{\infty} \frac{2n+1}{3n^2+2}(x-1)^{3n}$.

(iii) $\sum_{n=0}^{\infty} 3^{n^2} x^{n^2}$.

9.2. Prove that if $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$, then the radius of convergence of the series $\sum c_n (x-a)^n$ equals $1/c$.

9.3. Compute $\sum_{n=0}^{\infty} n(0.5)^n$.

9.4. Find a power series representation (centred at $x = 0$) of the function

$$f(x) = \frac{x^2}{(1-2x)^2}$$

and find its radius of convergence.

9.5. Find the Taylor series for

(i) $f(x) = xe^{2x}$ centred at $a = 0$

(ii) $f(x) = \frac{1}{x^2}$ centred at $a = 1$.

(iii) $f(x) = \ln(1 + x^2)$ centred at $a = 0$.

9.6. Suppose that the function $f(x)$ can be represented by a power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n}.$$

Find the first two terms of the Taylor series of $f(x)$ centred at $x = 0$. (Hint: use Problem 9.3).