COMPLEX ANALYSIS I, MATH 4156/9056, WINTER 2010

HOMEWORK ASSIGNMENT 1.

Due January 21.

1.1. Find the number of zeros of the polynomial $p(z) = z^9 + 2z^5 - 2z^4 + z + 3$ in the right half-plane.

1.2. Let $f(z)$ be an analytic function on the open unit disc $\mathbb{D} = \{|z| < 1\}$. Suppose there is an annulus $U = \{r < |z| < 1\}$ such that the restriction of $f(z)$ to U is one-to-one. Show that $f(z)$ is one-to-one on $\mathbb{D}$.

1.3. Suppose that f is holomorphic in $\operatorname{Im} z \geq 0$, attains real values on the x-axis, and bounded. Prove that $f = \text{const}$.

1.4. Suppose $f : \mathbb{D} \to \mathbb{D}$ is holomorphic, and

$$f(0) = f'(0) = \cdots = f^{(k-1)}(0) = 0.$$

Prove that $|f(z)| \leq |z|^k$, for all $z \in \mathbb{D}$.

1.5. (For 9056 only.) Let $f(z)$ be meromorphic in the unit disc $\mathbb{D}$ and holomorphic in a neighbourhood of the unit circle $\partial\mathbb{D} = \{|z| = 1\}$. Prove that for any number A such that

$$|A| > \max_{z \in \partial\mathbb{D}} |f(z)|$$

the number of points in $\mathbb{D}$ where f attains value A equals the number of poles of f in $\mathbb{D}$.