COMPLEX ANALYSIS I, MATH 4156/9056, WINTER 2010

HOMEWORK ASSIGNMENT 4.

Due March 11

4.1. Does there exist a holomorphic map from the unit disc onto $\mathbb{C}$?

4.2. Find a conformal map from the strip
$$\{-2010\,\pi < \operatorname{Re} z < 2010\,\pi\}$$
onto the unit disc.

4.3. (a) Give a complete description of all biholomorphic maps of $\mathbb{C} \setminus \{0\}$ onto itself.
(b) Now let $\Omega = \mathbb{C} \setminus \{p_1, p_2, \dots, p_n\}$, where $p_j \in \mathbb{C}$. Give an explicit description of biholomorphic automorphisms of Ω.

4.4. Suppose that $\{f_\alpha\}$ is a family of holomorphic functions on a domain Ω such that any sequence of functions in the family has a subsequence that converges normally on Ω. Prove that the family $\{f'_\alpha\}$ has the same property.

4.5. (For 9056 only.) Prove the following version of the Reflection Principle. Let γ be a smooth real analytic curve given by the real analytic defining function
$$\tilde{r}(x, y) = r(z, \overline{z}) = \sum_{j,k \geq 0} a_{jk} z^j \overline{z}^k.$$
Suppose that γ' is another real analytic curve given by the defining function
$$\tilde{\rho}(x', y') = r(z', \overline{z}') = \sum_{j,k \geq 0} b_{jk} z'^j \overline{z}'^k.$$
Let $z' = f(z)$ be a conformal mapping defined in a neighbourhood of γ such that $f(\gamma) = \gamma'$. Then the following holds:
$$r(z, \overline{w}) = 0 \iff \rho(f(z), \overline{f(w)}) = 0.$$