# Calculus 1501B - Winter 2012 - Section 001
# Assignment # 1

- Due Friday, January $27^{th}$, by the end of class. **Late assignments will not be accepted.**
- You may hand in your assignment at any time prior to the due date, either in my office (MC 268 - slip it under the door if I am not there) or in class.
- Each question/question part is worth 5 marks.

1. Suppose that $f$ is defined as

$$f(x) = \begin{cases} x\ , & \text{if} \quad x < 1\ , \\ 0\ , & \text{if} \quad x = 1\ , \\ 2 - x\ , & \text{if} \quad x > 1\ . \end{cases}$$

   Prove, by contradiction, that $f$ does not attain a global maximum on $[0, 2]$.

2. Your friend observes that if $f(x) = |x|$, then there is no point $c \in (-1, 1)$ for which for which $f'(c) = 0$. He concludes that this invalidates Rolle's Theorem. Explain why your friend is mistaken.

3. Your friend observes that if $f(x) = \frac{x^2-1}{x-1}$, then there is no point $c \in (-1, 0)$ for which $f'(c) = 0$. She concludes that this invalidates Rolle's Theorem. Explain why your friend is mistaken.

4. Suppose that $f$ is continuous on the closed interval $[a, b]$. Prove that there exists a point $c \in (a, b)$ for which

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\,dx \;.$$

*Hint*: Define $F(z) = \int_a^z f(x)\,dx$ for $z \in [a, b]$ and use the Fundamental Theorem of Calculus to deduce relevant properties of $F$.

5. Suppose that $f$ is a non-constant function which satisfies the conditions of Rolle's Theorem on the interval $[a, b]$.

   (a) Sketch a graph to convince yourself that there should be a point $c \in (a, b)$ for which $f'(c) > 0$. I would recommend that you include a judiciously chosen secant line to graphically illustrate how one might actually *locate* a point with the desired properties.

   (b) Prove that there exists a point $c \in (a, b)$ for which $f'(c) > 0$.

   (c) **Not for marks**: Why is it necessary to insist that $f(a) = f(b)$ in order to ensure the existence of a point with the desired properties? That is, why didn't this question read "Suppose that $f$ is a . . . conditions of the Mean Value Theorem . . ."?

6. A friend claims to have discovered a function $f$ which is continuous on $[-1, 3]$, differentiable on $(-1, 3)$, satisfies $f(-1) = 1$ and $f(3) = 4$, and obeys the inequality $f'(x) \leq \frac{1}{2}$ for all $x \in (-1, 3)$. Prove that this friend is mistaken (or worse, lying).

7. Evaluate the following integrals

(a) $$\int [\ln x]^3 \, dx$$

(b) $$\int \frac{2x^2 + 3x + 2}{x^3 + 2x^2 + x} \, dx$$

(c) $$\int_0^1 x^2 \tan^{-1} x \, dx$$

(d) $$\int \frac{x^2 + x + 3}{x^3 + x^2 + x} \, dx$$

8. Suppose that $f$ is a twice differentiable function. Find an antiderivative of $xf''(x)$.