

## CALCULUS 1501 WINTER 2012

### HOMEWORK ASSIGNMENT 1.

Due January 19.

- 1.1. Prove that the function  $f(x) = x^3$  is differentiable everywhere.
- 1.2. Formulate and prove a statement similar to Lemma 1.8 for the case when  $f'(x_0) < 0$ . (See [Lecture 1 of the Course Notes](#)).
- 1.3. Give example of a function which is continuous on the interval  $(-\infty, 0]$  but has neither a global maximum nor a global minimum value.
- 1.4. Prove that if a nonconstant function  $f(x)$  satisfies the conditions of Rolle's theorem on the interval  $[a, b]$ , then there exist points  $x_1$  and  $x_2$  on the interval  $(a, b)$  such that  $f'(x_1) < 0$  and  $f'(x_2) > 0$ .
- 1.5. On the interval  $(0, 2)$  there exists a point  $c$  such that the tangent line to the graph of the function  $y = x^3$  at the point  $(c, c^3)$  is parallel to the straight line passing through the points  $(0, 0)$  and  $(2, 8)$ .
  - (i). Explain without calculations why such point  $c$  necessarily exists.
  - (ii). Find  $c$ .
- 1.6. Prove using the Mean Value Theorem:  $\frac{x}{1+x} < \ln(1+x)$ , for  $x > 0$ .
- 1.7. Show that the equation  $x^4 + 4x + c = 0$  has at most two real roots. Here  $c$  is an arbitrary constant. (*Hint: argue by contradiction - suppose that there are three different roots. Now try to use Rolle's theorem.*) [This is Problem 20 in Section 4.2 of the textbook (p. 289).]