CALCULUS 1501 WINTER 2012

HOMEWORK ASSIGNMENT 3.

Due February 9.

3.1. Evaluate

$$\int \frac{2x^4+5x^2-2}{2x^3-x-1}dx$$

3.2. Write out the form of the partial fraction decomposition of the function

$$P(t)=\frac{t}{(t^2-1)^2(t^2+1)^3}$$

Do not determine the numerical values of the coefficients.

3.3. Determine whether the following improper integrals converge or diverge. Evaluate the integral if it converges.

(i) $\displaystyle\int_2^\infty \frac{dx}{x^2-1}$

(ii) $\displaystyle\int_0^\infty \cos 2x dx$

(iii) $\displaystyle\int_1^\infty \frac{\arctan x}{x^2}dx$

3.4. Use the identity

$$\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}$$

to evaluate $\Gamma(1/2)$ and $\Gamma(5/2)$. For relevant definitions see Lecture 3 of the Course Notes.

3.5. Using only the ϵ-N definition of convergence of a sequence prove

$$\lim_{n\to\infty}\frac{2n+1}{3n+2}=\frac{2}{3}.$$

3.6. Recall that the Fibonacci sequence is defined by

$$f_1=f_2=1,\quad f_n=f_{n-1}+f_{n-2},\quad \text{for } n>2.$$

Consider a sequence

$$s_1=1,\quad s_n=\frac{f_{n+1}}{f_n}\ \text{for } n>1.$$

Assume that s_n converges. Find its limit.

3.7. Let $\{s_n\}$ be defined as

$$s_1=0.3,\ s_2=0.33,\ s_3=0.333,\ldots$$

Prove that $\{s_n\}$ converges.