CALCULUS 1501 WINTER 2012

HOMEWORK ASSIGNMENT 5.

Due March 15.

5.1. Determine whether the series converges absolutely, conditionally, or diverges.

(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+10}$

(ii) $\displaystyle\sum_{n=1}^{\infty} n^5\left(\frac{2-3n}{4n+3}\right)^n$

(iii) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin 5n}{n^5}$

(iv) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(\ln n)^p}$, $p > 0$. (Determine for which p the series converges.)

5.2. Let $\{f_n\}$ be the Fibonacci sequence, given by $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$, for $n > 2$. Use Problem 3.6 (Homework 3) to assess the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{f_n}$$

5.3. Determine whether the series below converges absolutely, conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n\, n!}{n^n}$$

5.4. Assess the convergence of the following series

$$\sum_{n=2}^{\infty}\left(\frac{1}{\sqrt{n}-1}-\frac{1}{\sqrt{n}+1}\right).$$

5.5. Find the radius and the interval of convergence of the following power series

(i) $\displaystyle\sum_{n=0}^{\infty}5^n x^{3n}.$

(ii) $\displaystyle\sum_{n=0}^{\infty}\frac{2n+1}{3n^2+2}(x-1)^{3n}.$

(iii) $\displaystyle\sum_{n=0}^{\infty}3^{n^2}x^{n^2}.$

5.6. Prove that if $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$, then the radius of convergence of the series $\sum c_n(x-a)^n$ equals $1/c$.

5.7. Compute $\sum_{n=0}^{\infty} n\left(\frac{1}{3}\right)^n$.

5.8. Find a power series representation (centred at $x = 0$) of the function

$$f(x) = \frac{x^2}{(1-2x)^2}$$

and find its radius of convergence.