Practice Midterm #2

Questions Drawn from 2010 Tests/Exams

1. Consider the sequence given recursively by

$$a_1 = 2\,, \qquad\qquad a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right).$$

(a) Calculate a_2 and a_3.

(b) Assume that $\{a_n\}$ converges, and that $\lim_{n\to\infty} a_n = L$. Find L.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

converges or diverges. If it converges, find its sum.

3. (a) State the Monotone Sequence Theorem.

(b) Suppose $a_n > 0$ for $n = 1, 2, \ldots$, and that

$$s_n = \sum_{k=1}^{n} a_k \quad < \quad 2 - \frac{1}{n}$$

for $n > 1$. Prove that $\sum_{n=1}^{\infty} a_n$ converges.

4. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^3 + n^2 - 3}{\sqrt{n^9 + 7n^3 + 3n}}$$

converges or diverges. Justify your answer using an appropriate test.

5. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$$

converges or diverges. Justify your answer using an appropriate test.

6. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n + 4^n}{3^n}$$

converges or diverges. Justify your answer using an appropriate test.

7. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^{4n}}{(n!)^n}$$

converges or diverges. Justify your answer using an appropriate test.

8. Approximate the sum of the (convergent) series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n)!}$$

to within an error of 0.01. Leave your answer as the sum of fractions.

9. (a) Define what it means for $\sum_{n=1}^{\infty} a_n$ to be conditionally convergent.

(b) Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$$

is conditionally convergent, absolutely convergent or divergent. Justify your answer using one or more appropriate tests.

10. Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n!)^2}{(2n)!}$$

is conditionally convergent, absolutely convergent or divergent. Justify your answer using one or more appropriate tests.

11. Prove: If $a_n > 0$ for all $n \geq 1$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ converges.

12. Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,(x-4)^n}{n\,3^n}\,.$$

13. Express the function $f(x) = \dfrac{x}{1-x^2}$ as a power series. Be sure to indicate the radius and interval of convergence.