Practice Midterm #2

Questions Drawn from 2010 Tests/Exams

1. Evaluate $\displaystyle\int_0^1 x\tan^{-1}\left(x^2\right)\,dx$.

2. Evaluate $\displaystyle\int e^x \sin x\,dx$.

3. Evaluate $\int \frac{x^2 + x}{(x-1)^3}\, dx$.

4. Evaluate $\displaystyle\int \frac{2x^2 + x + 5}{x\,(x^2 - 2x + 5)}\, dx.$

5. Prove, using the Mean Value Theorem: If $x < 0$, then $e^x > 1 + x$.

6. Suppose that f is continuous on $[1, 3]$ and differentiable on $(1, 3)$. Further suppose that $f(1) = 7$ and $f'(x) < 1$ for all $x \in (1, 3)$. Prove, using the Mean Value Theorem, that $f(3) < 9$.

7. (a) Recall that the Gamma function is defined as $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$.
Using this definition, evaluate $\Gamma(2)$.

(b) Determine the value of the (convergent) improper integral $\int_0^\infty t^{30}e^{-t}\,dt$.

8. Use the Comparison Theorem for improper integrals to determine the convergence of

$$\int_0^1 \frac{1}{\sqrt{x^3 + x}}\, dx \quad .$$

If the integral converges, you do not need to evaluate it.

9. Consider the sequence given recursively by

$$a_1 = 2\,, \qquad\qquad a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right).$$

(a) Calculate a_2 and a_3.

(b) Assume that $\{a_n\}$ converges, and that $\lim_{n\to\infty} a_n = L$. Find L.

10. Determine whether the sequence given by

$$a_n = \frac{[\ln(n)]^2}{n}$$

converges or diverges. If it converges, find its limit.

11. (a) State the $\epsilon - N$ definition of $\lim_{n\to\infty} a_n = L$.

(b) Prove, using the definition asked for in part (a), that

$$\lim_{n\to\infty} \frac{1}{\ln n} = 0$$