Practice Midterm #1

Questions Drawn from 2011 Tests/Exams

1. Using the definition of the limit, prove that

   (a) $\lim_{n\to\infty} \frac{2^n}{1+2^n} = 1.$

   (b) $\lim_{n\to\infty} \sqrt{n^2-1} = \infty.$

2. Determine whether the following sequences are convergent or divergent. If a sequence is convergent, find its limit. Justify your answers.

   (a) $a_n = \dfrac{\ln(n)}{\sqrt{n}}$.

   (b) $a_n = \dfrac{\sin((-2)^n)}{n^3+1}$

   (c) $a_n = (-1)^n \dfrac{n}{n+1}$

3. (a) Give an example of a sequence which is bounded, but not convergent.

(b) Give an example of a sequence which is not bounded above and not bounded below.

4. Evaluate the integrals

(a) $\int \frac{\ln x}{x^2}\, dx.$

(b) $\int \frac{2x+1}{x^2+4x+13}\, dx.$

5. Evaluate $\displaystyle\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2}\, dx.$

6. Determine whether the improper integral $\int_0^2 \frac{2x}{\sqrt{4-x^2}}\, dx$ converges or diverges.

7. Determine whether the improper integral $\int_1^\infty \frac{1}{\sqrt{x+x^3}}\,dx$ converges or diverges.

8. Suppose that f is a function differentiable on $(0, \infty)$ and continuous on $[0, \infty)$, such that $f(1) = 1$ and $f'(x) > 1$ for all $x > 1$. Prove that $f(x) > x$ for all $x > 1$.

9. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence satisfying $a_n \geq 1/2$ for all n, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$. Use the definition of divergence to infinity directly to prove that $\lim_{n\to\infty} a_n = \infty$.