Practice Final Exam

Questions Drawn from 2011 Tests/Exams

1. Determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!\,(x-2)^n}{1 \cdot 3 \cdot \ldots \cdot (2n-1)} .$$

2. (a) Find the Maclaurin series for the function $f(x) = \dfrac{x}{(1-x)^2}$.

(b) Use your answer from (a) to evaluate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n5^n}{6^n}$ ..

3. (a) Find the first four non-zero terms in the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{1+x^3}}$.

(b) Find the Taylor series for $f(t) = e^{t/2}$, centered at $a = 1$.

4. Express $\int_0^1 \ln\left(1+x^2\right)\,dx$ as the sum of an infinite series of real numbers.

5. Let $f(x) = x \sin\left(\dfrac{x^2}{4}\right)$.

(a) Find the Maclaurin series for f.

(b) Compute $f^{(11)}(0)$ and $f^{(16)}(0)$.

6. Find the Taylor series for $f(x) = \ln(x)$, centered at $a = 2$. Find all values of x for which $f(x)$ is equal to the sum of the series.

7. Let C be the polar curve defined by $r = 2\sin(2\theta)$ for $\theta \in [0, 2\pi]$.

(a) Sketch C, labelling petals in the order with which they are traced out.

(b) Determine the area enclosed by one petal.

(c) Determine all (Cartesian) points where C intersects the curve defined by $x^2 + y^2 = 4$.

8. Let C be the parametric curve defined by the equations

$$x = t^2 \cos t\,, \qquad y = t^2 \sin t\,, \qquad t \in [0, 4\pi]\ .$$

(a) Sketch C, indicating the points of intersection with the coordinate axes.

(b) Determine the equation of the tangent line at the point $\left(0, -\frac{9\pi^2}{4}\right)$.

9. Solve the initial value problem

$$\frac{dx}{dt} + x\cos t = 5\cos t\,, \qquad x(\pi/2) = 10\ .$$

10. A tank contains 100 litres of water, in which 2 kilograms of salt is dissolved. Pure water enters the tank at a rate of 5 litres per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How long does it take for the concentration of salt in the tank to reach half of its original level?

11. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence satisfying $a_n \geq 1/2$ for all n, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$. Prove that $\lim_{n\to\infty} a_n = \infty$.