CALCULUS 1501 WINTER 2013

HOMEWORK ASSIGNMENT 1.

Due January 24.

1.1. Prove that the function $f(x) = x^3$ is differentiable everywhere.

1.2. Formulate and prove a statement similar to Lemma 1.8 for the case when $f'(x_0) < 0$. (See Lecture 1 of the Course Notes).

1.3. Give example of a function which is continuous on the interval $(-\infty, 0]$ but has neither a global maximum nor a global minimum value.

1.4. Prove that if a nonconstant function $f(x)$ satisfies the conditions of Rolle's theorem on the interval $[a, b]$, then there exist points x_1 and x_2 on the interval (a, b) such that $f'(x_1) < 0$ and $f'(x_2) > 0$.

1.5. On the interval $(0, 2)$ there exists a point c such that the tangent line to the graph of the function $y = x^3$ at the point (c, c^3) is parallel to the straight line passing through the points $(0, 0)$ and $(2, 8)$.

(i). Explain without calculations why such point c necessarily exists.

(ii). Find c.

1.6. Prove using the Mean Value Theorem: $\dfrac{x}{1+x} < \ln(1+x), \quad \text{for } x > 0.$

1.7. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots. Here c is an arbitrary constant. *(Hint: argue by contradiction - suppose that there are three different roots. Now try to use Rolle's theorem.)* [This is Problem 20 in Section 4.2 of the textbook (p. 289).]