CALCULUS 1501 WINTER 2013

HOMEWORK ASSIGNMENT 3.

Due February 28.

3.1. Using only the ϵ-N definition of convergence of a sequence prove

$$\lim_{n\to\infty} \frac{2n+1}{3n+2} = \frac{2}{3}.$$

3.2. Recall that the Fibonacci sequence is defined by

$$f_1 = f_2 = 1, \quad f_n = f_{n-1} + f_{n-2}, \quad \text{for } n > 2.$$

Consider a sequence

$$s_1 = 1, \quad s_n = \frac{f_{n+1}}{f_n} \text{ for } n > 1.$$

Assume that s_n converges. Find its limit.

3.3. Let $\{s_n\}$ be defined as

$$s_1 = 0.3, \; s_2 = 0.33, \; s_3 = 0.333, \ldots$$

Prove that $\{s_n\}$ converges.

3.4. Find the limit of the sequence

$$\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots \right\}$$

3.5. Determine convergence of the following series

(a) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$,

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^3}$,

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$.

3.6. Give example of a pair of series $\sum a_n$ and $\sum b_n$ with positive terms with the property that $\lim_{n\to\infty}(a_n/b_n) = 0$, and $\sum b_n$ diverges, but $\sum a_n$ converges.

3.7. Prove that if $a_n > 0$ and $\lim_{n\to\infty} n a_n \neq 0$, then $\sum a_n$ is divergent.

3.8. Find all positive values of b for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges.