

CALCULUS 1501 WINTER 2013

HOMEWORK ASSIGNMENT 5.

Due March 28.

- 5.1. Let $\{f_n\}$ be the Fibonacci sequence, given by $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$, for $n > 2$. Use Problem 3.2 ([Homework 3](#)) to assess the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{f_n}.$$

- 5.2. Find a power series representation (centred at $x = 0$) of the function

$$f(x) = \frac{x^2}{(1-2x)^2}$$

and find its radius of convergence.

- 5.3. Evaluate the integral

$$\int_0^1 \frac{\ln(1-x)}{x} dx.$$

Hint: Use Taylor series expansion and the identity $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

- 5.4. Find the arclength function for the curve $y = \frac{1}{3}x^3 + \frac{1}{4x}$ with starting point $P(1, \frac{7}{12})$.

- 5.5. Graph the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$. Find its length.

- 5.6. Find parametric equations for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $a > 0$, $b > 0$ are constants.

- 5.7. Find a Cartesian equation of the curve $x = \ln t$, $y = \sqrt{t}$, $t > 1$. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.