

MATH 9133 WINTER 2019.

HOMEWORK ASSIGNMENT 1. DUE FEB 5.

- 1.1. Prove that if a map $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is differentiable at a point p , then it is continuous there. Give example of a function that is continuous at the origin, has there partial derivatives, but is not differentiable.
- 1.2. Suppose that a differentiable function $f : \Omega \rightarrow \mathbb{R}$ on a domain Ω satisfies $Df(x) = 0$ for all $x \in \Omega$. Prove that f is a constant function.

- 1.3. Consider the function

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Show that $D_{xy}f \neq D_{yx}f$.

- 1.4. If f and g are differentiable real functions on \mathbb{R}^n , prove that

$$\nabla(fg) = f\nabla g + g\nabla f,$$

and that for $f \neq 0$,

$$\nabla(1/f) = -\frac{\nabla f}{f^2}.$$