# MATH 9133 WINTER 2019

HOMEWORK ASSIGNMENT 2. DUE FEBRUARY 26.

2.1. Let $X \subset \mathbb{R}^n$ be a measurable set. Prove that if a function $f : X \to \mathbb{R}$ vanishes outside a set of measure zero, then $\int_X f(x)dx = 0$ (Lebesgue integral). Start by proving that $f$ is measurable.

2.2. The Monotone Convergence theorem states that if $\{f_n\}$ is an increasing sequence of non-negative measurable functions and $f = \lim f$ a.e., then

$$\int f dx = \lim \int f_n dx.$$

Use this theorem to prove that given a nonnegative integrable $f$ on $\mathbb{R}$, the function

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

is continuous.

2.3. Let $C = C^0[0,1]$ be the space of all continuous function on $[0,1]$, and define

$$||f|| = \max_{x\in[0,1]} |f(x)|.$$

Show that $C$ is a Banach space.

2.4. (i) Prove that a linear functional $\phi$ on a topological vector space $X$ is continuous if and only if there exists a neighbourhood of the origin where $\phi$ is bounded.

(ii) Suppose that $X$ is an infinite dimensional normed space. Prove there exists a noncontinuous linear functional on $X$.