MATH 9133 WINTER 2019

HOMEWORK ASSIGNMENT 4. DUE APRIL 2.

4.1. Suppose (f_k) is a sequence of continuous functions converging to a function f in $\mathcal{C}(\mathbb{R})$. Prove that (f_k) converges to f in $\mathcal{D}'(\mathbb{R})$.

4.2. Prove that
$$\lim_{n\to\infty} \sin(nx) = 0$$
in $\mathcal{D}'(\mathbb{R})$, but
$$\lim_{n\to\infty} \sin^2(nx) \neq 0.$$
Conclude that multiplication of distributions is not a continuous operation even when it is defined.

4.3. Suppose that f and g are test functions on $\mathbb{R}^n$. Prove that $f * g = g * f$

4.4. Consider a linear functional on $\mathcal{D}(\mathbb{R})$ defined by

$$\langle f, \varphi \rangle = \sum_{n=0}^{\infty} \varphi^{(n)}(n).$$

Prove that f is not of finite order.